$$[(1+\xi)^2P_{n1}+y^2/x^2P_{n1}']\cdot\left[\left(1+\frac{r_1}{r_2}\frac{\cot\psi_2}{\cot\psi_1}\xi\right)^2P_{n2}+\left(\frac{\cot\psi_2}{\cot\psi_1}\right)^2y^2/x^2P_{n2}'\right]$$

$$-\left[\left(1+\frac{r_1}{r_2}\frac{\cot\psi_2}{\cot\psi_1}\xi\right)^2R_n+\frac{\cot\psi_2}{\cot\psi_1}y^2/x^2R_n'\right]$$

$$\cdot\left[(1+\xi)\left(1+\frac{r_1}{r_2}\frac{\cot\psi_2}{\cot\psi_1}\xi\right)R_n+y^2/x^2\frac{\cot\psi_2}{\cot\psi_1}R_n'\right]=0. \quad (8)$$

in a more compact form by defining the following notation:

$$A_1=\frac{n\beta}{\tau^2r_1}\quad A_2=\frac{n\beta}{\tau^2r_2}\qquad B=\frac{i\omega\epsilon}{\tau}$$

$$c=\frac{i\mu\omega}{\tau}\quad I_{n1}=I_n(\tau r_1)\quad K_{n2}=K_n(\tau r_2),\text{ etc.}$$

Writing the equation with this notation and removing the common factor, F_n, we have:

$$I_{n1}(1+A_1\cot\psi_1)a_n^{(1)}+c\cot\psi_1I_{n1}'b_n^{(1)}=0 \quad (7a)$$

$$A_1I_{n1}a_n^{(1)}+cI_{n1}'b_n^{(1)}-A_1I_{n1}a_n^{(2)}-cI_{n1}'b_n^{(2)}-A_1K_{n1}c_n^{(2)}-cK_{n1}'d_n^{(2)}=0 \quad (7b)$$

$$I_{n1}a_n^{(1)}-I_{n1}a_n^{(2)}-K_{n1}c_n^{(2)}=0 \quad (7c)$$

$$B\cot\psi_1I_{n1}'a_n^{(1)}-I_{n1}(1+A_1\cot\psi_1)b_n^{(1)}-B\cot\psi_1I_{n1}'a_n^{(2)}+I_{n1}(1+A_1\cot\psi_1)b_n^{(2)}-B\cot\psi_1K_{n1}'c_n^{(2)}+K_{n1}(1+A_1\cot\psi_1)d_n^{(2)}=0 \quad (7d)$$

$$K_{n2}(1+A_2\cot\psi_2)a_n^{(3)}+c\cot\psi_2K_{n2}'b_n^{(3)} \quad (7e)$$

$$A_2I_{n2}a_n^{(2)}+cI_{n2}'b_n^{(2)}+A_2K_{n2}c_n^{(2)}+cK_{n2}'d_n^{(2)}-A_2K_{n2}a_n^{(3)}-cK_{n2}'b_n^{(3)}=0 \quad (7f)$$

$$I_{n2}a_n^{(2)}+K_{n2}c_n^{(2)}-K_{n2}a_n^{(3)}=0 \quad (7g)$$

$$B\cot\psi_2I_{n2}'a_n^{(2)}-I_{n2}(1+A_2\cot\psi_2)b_n^{(2)}+B\cot\psi_2K_{n2}'c_n^{(2)}-K_{n2}(1+A_2\cot\psi_2)d_n^{(2)}-B\cot\psi_2K_{n2}'a_n^{(3)}+K_{n2}(1+A_2\cot\psi_2)b_n^{(3)}=0. \quad (7h)$$

In order to find the permissible values of τ we must set the system determinant equal to zero. The expansion of the eighth order determinant is carried out elsewhere.[3] In order to write the determinantal equation in a compact form we define

$$P_{n1}=I_{n1}K_{n1},\qquad P_{n1}'=I_{n1}'K_{n1}',$$
$$R_n=I_{n1}K_{n2},\qquad R_n'=I_{n1}'K_{n2}',$$
$$P_{n2}=I_{n2}K_{n2},\qquad P_{n2}'=I_{n2}'K_{n2}',$$
$$x=\tau r_1,\qquad y=kr_1\cot\psi_1,$$

and

$$\xi=n\frac{\sqrt{\cot^2\psi_1x^2+y^2}}{x^2}\cdot\frac{\cot\psi_1}{|\cot\psi_1|}.$$

The general determinantal equation is then found to be

[3] R. E. Hayes, "A Study of Coupled Helices," M.S. thesis, University of Kansas, Lawrence; May, 1959.

The values of τ which satisfy (8) give the propagation constants of the various modes of propagation. Setting $n=0$ in (8) we obtain the previously known expression for the lowest mode.[1] This mode is commonly used in the analysis of helical couplers for traveling-wave tubes. Letting r_2 approach infinity we obtain the equation for the modes on a single helix.[4] These special cases indicate the correctness of (8).

The solutions of (8) were found by an approximate method using a digital computer.[3] The modes of propagation for $n=0$, ±1 are shown in Fig. 2 and Fig. 3. The curves for $n=0$ were calculated from published data,[1] while Fig. 3 is the result of the computer solution. Similar curves may be found for any value of n. The knowledge of these higher order modes should make it possible to obtain more accurate solutions to the coupled helix problem.

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[4] S. Sensiper, "Electromagnetic Wave Propagation in Helical Conductors," Res. Lab. Electronics, Mass. Inst. Tech., Cambridge, Mass., Tech. Rep. No. 194, p. 7; May, 1951.

The Tetrahedral Junction as a Waveguide Switch*

A junction of two rectangular waveguides which are mutually cross-polarized becomes a magnetically controlled reactive switch when properly loaded by a ferrite rod magnetized longitudinally (see Fig. 1). It is a special case of a novel type of structure for which we propose the name *tetrahedral junction*. As a switch, it possesses:

1) very high insertion loss in the reflecting state, ~60 db;
2) loss in the transmitting state which is lower in principle than that attainable in any similar ferrite-waveguide device, <0.1 db;
3) high switching speed—1 μsec is attainable with conventional circuits and convenient currents;
4) large bandwidth, ~10 per cent;
5) little sensitivity to variations in applied field and saturation magnetization; and
6) small phase and small phase-variations with frequency and applied field in the transmitting state.

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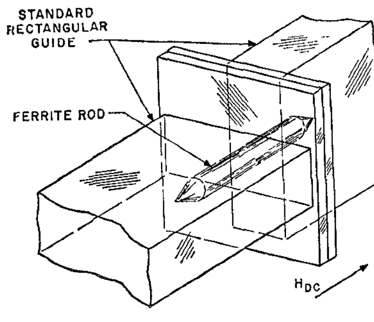

Fig. 1—The tetrahedral junction.

Our experimental and theoretical results indicate that there is room for further improvement in most of its significant properties.

In addition to its utility as a switch, the tetrahedral junction can also be used as a reversible gyrator. Further, its principle of operation is such that a dissipative element can be incorporated which converts the device into a matched modulator or reversible isolator.

The name tetrahedral derives from the fact that if the ends of the two crossed guides are separated and their parallel edges joined by planes, the resulting taper is in the form of a doubly-truncated tetrahedron. Some of our models are of this form, in particular, the one whose properties are reported above.

Our interest in this device is an outgrowth of our study of the Reggia-Spencer phase shifter;[1,2,3] it is one of the "novel effects" to which allusion is made by Weiss.[3] As in the case of the phase shifter, the behavior of the tetrahedral junction may be divided into two regimes: above a sharply defined frequency the device takes on the properties of a Faraday rotator; in a frequency range just below that of the Faraday effect regime, the junction exhibits its most interesting and useful characteristics. Here, however, it is inappropriate to speak of the phase shift regime, for the phase shift effects which are central to the Reggia-Spencer device are inessential, in fact undesirable, in this case. On the other hand, the modes of propagation[3] are of the same basic form in the two. In the presence of the magnetized ferrite, a wave entering the junction of Fig. 1 from the input end takes on a characteristic elliptic polarization. At the plane of the joint it is scattered into four significant components: a reflected and a transmitted propagating mode, and an evanescent mode in each guide which is also elliptically polarized. A model of this phenomenon, employing simplifying assumptions similar to those used by Weiss,[3] shows that under the proper conditions (involving the cross-sectional dimensions of the two guides, and the

[1] F. Reggia and E. G. Spencer, "A new technique in ferrite phase-shifting for beam scanning of microwave antennas," Proc. IRE, vol. 45, pp. 1510–1517; November, 1957.

[2] J. A. Weiss, "The Reggia-Spencer microwave phase shifter," *J. Appl. Phys.*, vol. 30, pp. 153s–154s; April, 1959. *Proc. AIEE Conf. on Magnetism and Magnetic Materials*, Philadelphia, Pa., November, 1958.

[3] J. A. Weiss, "A phenomenological theory of the Reggia-Spencer phase shifter," Proc. IRE, vol. 47, pp. 1130–1137; June, 1959.

diameter and magnetic parameters of the ferrite) the incident wave is fully matched into the transmitted wave. The only portion of the ferrite which contributes in an essential way to the effect is that located in the immediate vicinity of the joint; thus there is, so to speak, *no length* of active material involved in the principle. It is therefore possible to use only a small ferrite volume, with the advantage, among others, of minimizing the magnetic loss. When the ferrite is unmagnetized, the junction simply presents a transition to cutoff guide; the insertion loss available in this state is limited only by the precision of the mechanical structure.

The phase, insertion loss, and polarization in the transmitting state exhibit a number of well-defined features which agree qualitatively with the results of the simplified theoretical model. The latter, in turn, indicates how the effect may be exploited to fulfill the various specific application requirements. A full report on the theory and experimental data will be the subject of a forthcoming paper.

By way of acknowledgment, I wish to thank my colleagues for their interest and comments. I am particularly indebted to R. W. Judkins for his contributions to the various aspects of this work.

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Some Measurements of Traveling-Wave Tube Attenuators at 2000 MC*

Saturation in traveling-wave tube amplifiers, that is, the failure of the output to continue to rise as the input power is increased, can occur in several ways. The limit may arise through heating of the RF circuit, or if the circuit can be so made that this does not occur, the ultimate limit is provided by the amount of power that can be carried on the beam; large signal calculations of the behavior under these conditions have been made by several investigators.[1–4] It has also been observed, however, that the saturation limit is a function of the amount and distribution of loss in the attenuator, and recent calculations, for example by Rowe,[4] include the study of the limiting power output for various values of the loss parameter *d*. However, as pointed out by Caldwell,[5] the saturation limit in the attenuator may depend at least as much on any modification of the phase velocity in the attenuating region as on the amount of loss. This arises because in nearly all practical tubes the attenuator does not conform to either of the two ideal cases; *i.e.*, it is not completely distributed throughout the length of the circuit or concentrated at one point, but occupies an appreciable part of the total length. As a result of this, if the phase velocity of the circuit wave in the attenuating zone is modified, there is a reduced interaction between the circuit wave and the beam, since the beam velocity is arranged to give optimum interaction with the wave on the loss-free part of the circuit.

It is therefore of interest to know how the attenuation and phase velocity vary in practical attenuators, and the first object of the work described here is to relate these quantities to the surface resitivity of "Aquadag" colloidal graphite material coatings on the surface of ceramic rods, as has been done by Caldwell[5] for external sprayed attenuation.

Some measurements have therefore been made at 2000 mc using steatite rods which were sprayed with Aquadag mixture along the whole of the helix length. The coating surface resistivity was measured by the use of a four-terminal resistance bridge, from which current was passed through the coating between thin wire loops drawn around the outside, and the potential difference measured between two points 3 mm apart near their center. The resistivity was found to vary along the length of the coating by as much as 35 per cent, but since the total range investigated covered 8 decades the variations were not so great that a mean value could not be sensibly used (see Fig. 1).

The rods were then placed around a helix, which was mounted in a well-matched circuit, and the insertion loss was measured.

The phase velocity measurements were made by passing an external coupling loop along the length of the assembled helix, which for this purpose had a solid mandrel screwed into the end remote from the RF feed so that a large standing-wave was set up. The arrangement used is shown diagrammatically in Fig. 2. Because of the amount of attenuation present, and the need to avoid errors due to evanescent waves arising from the discontinuities at the ends, these measurements became progressively more difficult as higher surface conductivities were used. A better way of making the measurements would be to assemble the rods into sealed-off traveling-wave tubes, and to find the electron beam velocity for minimum attenuation at very low beam currents. This has not been possible because of the magnitude of the work involved.

Typical standing-wave patterns obtained are well shown in Figs. 3–6. Fig. 3 shows the pattern with no coating of "Aquadag" colloidal graphite material on the rods. It

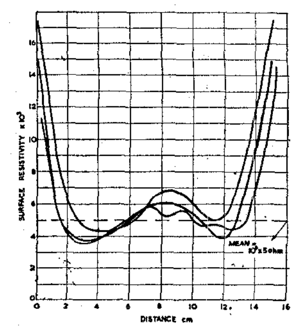

Fig. 1—Variation of surface resistivity with length.

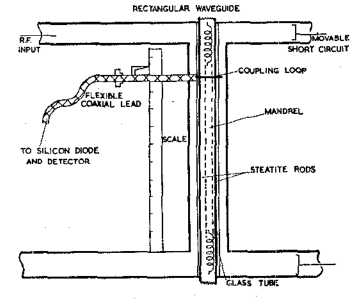

Fig. 2—Arrangement of pick-up loop.

may be seen that the pattern is distorted by the existence of evanescent fast-waves set up at the waveguide coupling, which could also account for the apparent increase of signal with distance shown in Figs. 5 and 6. The attenuation measurements were, of course, not made using this circuit, but instead a well-matched close-fitting circuit was used, in which only the slow mode was present. The surface resistivity in this case was taken as 10^8 ohms so that a representative point for the "no loss" case could be included in the curves. Fig. 4 shows the pattern for rods with a surface resitivity of 8×10^5 ohms. The wavelength measurements were taken over the four central minimas, since these were found to be uniformly spaced. In Fig. 5, the resistivity has been reduced to 5.9×10^3 ohms, and in Fig. 6 to 7.2×10^2. The lowest resistance coatings that it was possible to measure in this way had a surface resistivity of 3.4×10^2 ohms, which gave a total attenuation of 107 db, and a wavelength of 4 mm compared with the wavelength of 1.12 cm obtained for the uncoated rods. The results obtained from these measurements are shown graphically in Fig. 7. The measurements were carried out at 2000 mc, using a helix with a pitch of 1.5 mm, and for which $\gamma a_0 = 1.3$ when mounted between steatite rods and enclosed in a glass tube. The rod diameter was nearly 0.3 times the mean helix diameter.

The curves obtained by Caldwell for externally sprayed attenuators of "Dixonac" colloidal graphite material for use in the 500 mc region are shown for comparison in Fig. 8.

It is interesting to note that the general form of the curves is similar, although the values of attenuation are much lower for the 2000 mc curves, and the effect of resistive coatings on the phase velocity is apparently greater. This similarity is more surprising when it is borne in mind that in

* Received by the PGMTT, July 10, 1959; revised manuscript received, September 14, 1959.

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[2] P. K. Tien, L. R. Walker and V. M. Wolontis, "Large signal theory of traveling-wave amplifiers," Proc. IRE, vol. 43, pp. 260–277; March, 1955.

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